Transmission lines in MMIC technology

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Transmission lines in MMIC technology

- Propagation in transmission lines
- Topologies of transmission lines
- Planar transmission lines
  - Microstrip line
  - Coplanar waveguide
- Coaxial cable
- Waveguides
Transmission lines in MMIC technology

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Transmission lines in MMIC technology

- Propagation in transmission lines

  - Definition of propagation modes:

    - **TE modes** (Transverse Electric) have no electric field in the direction of propagation.
    - **TM modes** (Transverse Magnetic) have no magnetic field in the direction of propagation.
    - **TEM modes** (Transverse ElectroMagnetic) have no electric nor magnetic field in the direction of propagation.
    - **Hybrid modes** have both electric and magnetic field components in the direction of propagation.

Generally, transmission lines are used respecting certain conditions in order to only have TEM modes of propagation.
Transmission lines in MMIC technology

- Propagation in transmission lines
  - At **low frequency**:
    - At one time "t", the points of the circuit linked by a conductor are at the same potential:
      \[(V_A - V_B)(t) = (V_C - V_D)(t)\]
Transmission lines in MMIC technology

- Propagation in transmission lines
  - At **high frequency**: When the frequency “f” of the signal increases enough, the differences of potential between two points linked by a conductor have not equal values any more:

\[
(V_A - V_B)(t) \neq (V_C - V_D)(t)
\]
Transmission lines in MMIC technology

- Propagation in transmission lines

  - At high frequency:
    - We can define the **wavelength** of the signal which propagates at the frequency “f” by the following expression:
      \[ \lambda = \frac{c}{f} (m) \]

      where c is the speed of the light in vacuum:
      \[ c = 3 \times 10^8 (m/s) \]

- Examples of wavelengths at different frequencies:

<table>
<thead>
<tr>
<th>Frequency f</th>
<th>Wavelength ( \lambda )</th>
</tr>
</thead>
<tbody>
<tr>
<td>50 Hz</td>
<td>6000 km</td>
</tr>
<tr>
<td>5 kHz</td>
<td>60 km</td>
</tr>
<tr>
<td>500 kHz</td>
<td>600 m</td>
</tr>
<tr>
<td>5 GHz</td>
<td>6 cm</td>
</tr>
<tr>
<td>50 GHz</td>
<td>6 mm</td>
</tr>
</tbody>
</table>
Transmission lines in MMIC technology

- Propagation in transmission lines:
  - Each time the wavelength is in the same order of the physical dimensions of the conductor linking to points, it is necessary to take into account the propagation effects:

  \[ \text{If } \ell \approx \lambda \Rightarrow \text{propagation effects!} \]

  Electrical length \(\neq\) Physical length!

  \(\Rightarrow\) Theory of the transmission lines
Transmission lines in MMIC technology

- Propagation in transmission lines:
  - **Equivalent circuit of a transmission line** with a TEM (or quasi-TEM) propagation:
    - A transmission line is a structure with distributed elements (due to propagation effects),
    - The elements of the equivalent circuit are given in units of length:
      - Resistance: $R$ (Ω/m),
      - Inductance: $L$ (H/m),
        \[
        \{ \text{Associated to the conductors} \}
        \]
      - Capacitance: $C$ (F/m),
      - Conductance: $G$ (S/m = $\Omega^{-1}$/m).
        \[
        \{ \text{Associated to the substrate} \}
        \]
    - $R \Rightarrow$ conductor losses!
    - $G \Rightarrow$ substrate losses!
Transmission lines in MMIC technology

- Propagation in transmission lines:
  - Two equivalent circuits for transmission lines:
    - **Line without losses:**
      \[ Z = jL \omega \]
      \[ Y = jC \omega \]
    - **Line with losses:**
      \[ Z = R + jL \omega \]
      \[ Y = G + jC \omega \]
Transmission lines in MMIC technology

- Propagation in transmission lines:
  - Equivalent circuit of a transmission line with a TEM (or quasi-TEM) propagation:
    - Coupled differential equations $\Rightarrow$ Equation of telegraphers:
      \[
      \frac{\partial \bar{V}}{\partial x} = -(R + jL\omega)i \\
      \frac{\partial \bar{I}}{\partial x} = -(G + jC\omega)\bar{V}
      \]
    - By combining these two equations, we can obtain two independent equations:
      \[
      \frac{\partial^2 \bar{V}}{\partial x^2} = \gamma^2 \bar{V} \\
      \frac{\partial^2 \bar{I}}{\partial x^2} = \gamma^2 \bar{I}
      \]
    - Where $\gamma$ is the propagation constant:
      \[
      \gamma = \sqrt{(R + jL\omega)(G + jC\omega)} \\
      \gamma = \alpha + j\beta \\
      \gamma = \sqrt{Z\bar{Y}}
      \]
    - With $\alpha$ is the attenuation constant (Nepers/m) and $\beta$ is the phase constant (rad/m).
Transmission lines in MMIC technology

- Propagation in transmission lines:
  - Equivalent circuit of a transmission line with a TEM (or quasi-TEM) propagation:

    - The previous equations in voltage and current have as solutions a combinations of waves propagating on the transmission line in opposite directions:

      \[ V(x) = V_i e^{-j\gamma x} + V_r e^{j\gamma x} \]
      \[ I(x) = I_i e^{-j\gamma x} - I_r e^{j\gamma x} = \frac{1}{Z_c} \left( V_i e^{-j\gamma x} - V_r e^{j\gamma x} \right) \]

    - Where \( V_i \) and \( V_r \), and \( I_i \) and \( I_r \) are linked to the incident and reflected waves, respectively.

    - \( Z_c \) is the **characteristic impedance**:

      \[ Z_c = \frac{V_i}{I_i} = -\frac{V_r}{I_r} = \sqrt{\frac{R + j\omega L}{G + j\omega C}} = \sqrt{\frac{Z}{Y}} \]
Transmission lines in MMIC technology

- Propagation in transmission lines:
  - Incident and reflected waves:
    - From the previous expressions of voltage and current through the transmission lines:
      \[
      V(x) = V_i e^{(-j\kappa x)} + V_r e^{(j\kappa x)}
      \]
      \[
      I(x) = I_i e^{(-j\kappa x)} - I_r e^{(j\kappa x)} = \frac{1}{Z_c} \left( V_i e^{(-j\kappa x)} - V_r e^{(j\kappa x)} \right)
      \]
    - We can define the expressions in voltage and current of the incident and reflected waves:
      \[
      V_i(x) = V_i e^{(-j\kappa x)}
      \]
      \[
      I_i(x) = \frac{V_i}{Z_c} e^{(-j\kappa x)} \quad \text{Incident wave}
      \]
      \[
      V_r(x) = V_r e^{(j\kappa x)}
      \]
      \[
      I_r(x) = \frac{V_r}{Z_c} e^{(-j\kappa x)} \quad \text{Reflected wave}
      \]
Transmission lines in MMIC technology

- Propagation in transmission lines:
  - Reflection coefficient and impedance at each point of the transmission line:
    - Two systems of axis are considered:
      - Towards the load,
      - Towards the generator.
Transmission lines in MMIC technology

- Propagation in transmission lines:
  - Reflection coefficient and impedance at each point of the transmission line:
    - **Reflection coefficient** at each point of the transmission line:
      \[
      \Gamma(x) = \frac{V_r(x)}{V_i(x)} = \frac{V_r}{V_i} e^{i2\pi x}
      \]
    - **Impedance at each point** of the transmission line:
      \[
      Z(x) = \frac{V(x)}{I(x)} = \frac{V_r e^{-i\pi x} + V_i e^{i\pi x}}{1} = Z_c \frac{1}{Z_c} \frac{V_r e^{i2\pi x}}{V_i} = Z_c \frac{1+\Gamma(x)}{1-\Gamma(x)}
      \]
    - **Reduced impedance**:
      \[
      z(x) = \frac{Z(x)}{Z_c} = \frac{1+\Gamma(x)}{1-\Gamma(x)}
      \]
Transmission lines in MMIC technology

- Propagation in transmission lines:
  - Reflection coefficient and impedance at each point of the transmission line:
    - The reflection coefficient can be expressed from the impedance:
      \[ \Gamma(x) = \frac{Z(x) - 1}{Z(x) + 1} = \frac{Z(x) - Z_C}{Z(x) + Z_C} \]
    - At the abscissa \( x = L \), the impedance is equal to the load impedance \( Z_L \):
      \[ Z(x) = Z_L \]
    - Then:
      \[ \Gamma(L) = \frac{Z_L - Z_C}{Z_L + Z_C} = \Gamma_L \]
    - If \( Z_L = Z_C \) then the reflection coefficient is null (\( \Gamma = 0 \)) and the transmission line is matched.
Transmission lines in MMIC technology

- Propagation in transmission lines:
  - Reflection coefficient and impedance at each point of the transmission line:
    - In the plane of the load \((s=0)\), the reflection coefficient is:
      \[
      \Gamma_L = \Gamma(s = 0) = \frac{V_r}{V_i} e^{(0)} = \frac{V_r}{V_i}
      \]
    - We have seen that at each point of the transmission line:
      \[
      \Gamma(s) = \frac{V_r(s)}{V_i(s)} = \frac{V_r}{V_i} e^{(-2j\gamma s)}
      \]
    - So:
      \[
      \Gamma(s) = \Gamma_L e^{(-2j\gamma s)}
      \]
    - From these expressions, one can obtain the expression of the **reduced impedance brought back at the entry**: 
      \[
      z(s) = \frac{z_L + th(js)}{1 + z_L th(\gamma s)}
      \]
Transmission lines in MMIC technology

- Propagation in transmission lines:
  - Reflection coefficient and impedance at each point of the transmission line:
    - In the case of a transmission line without losses, the propagation constant is the following:
      \[ \gamma = j\beta \]
    - From the trigonometric properties, and replacing the propagation constant in the expression of the impedance, one can obtained, for the reduced impedance brought back at the entry:
      \[ z(s) = \frac{z_L + j\tan(\beta s)}{1 + jz_L \tan(\beta s)} \]
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- Propagation in transmission lines:
  - Stationary waves ratio:
    - Generally, the reflection coefficient is not null, they exist stationary waves along the transmission lines, and we can define the stationary waves ratio:
    \[
    SWR = \frac{|V_{\text{max}}|}{|V_{\text{min}}|} = \frac{1 + |\Gamma_L|}{1 - |\Gamma_L|}
    \]
Transmission lines in MMIC technology

- Propagation in transmission lines:
  - Transmission lines terminated by particular load:
    - Transmission lines terminated by a matched load: \( Z_L = Z_C \)
      - \( \Gamma_L = 0 \Rightarrow \) no reflection,
      - \( \text{SWR} = 1 \Rightarrow \) progressive wave,
      - \( z(s) = 1 \) or \( Z(s) = Z_C \Rightarrow \) the impedance at each point is equal to the characteristic impedance of the transmission line,
      - The wave propagates along the transmission line at the phase speed:
        \[ v_\phi = \frac{\omega}{\beta} \text{(m/s)} \]
Transmission lines in MMIC technology

- Propagation in transmission lines:
  - Transmission lines terminated by particular load:
    - Transmission lines terminated by a short-circuit: $Z_L = 0$
      - $\Gamma_L = -1 \Rightarrow$ total reflection,
      - $\text{SWR} = \infty$,
      - The impedance at each point of the transmission line is equal to:
        $$z(s) = jtg(\beta s)$$
Transmission lines in MMIC technology

- Propagation in transmission lines:
  - Transmission lines terminated by particular load:
    - Transmission lines terminated by an open-circuit: \( Z_L = \infty \)
      - \( \Gamma_L = 1 \Rightarrow \) total reflection,
      - \( \text{SWR} = \infty \),
      - The impedance at each point of the transmission line is equal to:
        \[
        z(s) = \frac{1}{jtg(\beta s)}
        \]
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  - Microstrip line
  - Coplanar waveguide
- Coaxial cable
- Waveguides
Transmission lines in MMIC technology

- Topologies of transmission lines:
  - Choice of the topology of a transmission line:
    - Frequency of the signal,
    - Technology employed,
    - Power to transmit,
    - …Application…
  - Lines with several conductors:
    - Twisted pair line,
    - Coaxial cable,
    - Planar lines.
  - Transmission with only one conductor:
    - Waveguides.
Transmission lines in MMIC technology

- Topologies of transmission lines:
  - Twisted pair line
  - Coaxial cable
  - Waveguides
Transmission lines in MMIC technology

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Transmission lines in MMIC technology

- Planar transmission lines:
  - Advantages of planar transmission lines:
    - technology of integrated circuit,
    - planar conductors,
    - little dimensions,
    - propagation with a TEM or quasi-TEM mode.
  - Structure of planar transmission lines:
    - Generally, one conductor supports the signal,
    - Another is the ground plane,
    - A (dielectric) substrate is used as mechanical support.
Transmission lines in MMIC technology

- Planar transmission lines:
  - Different topologies of planar transmission lines:
    - Microstrip line
    - Stripline
    - Coplanar line
    - Slotline
    - Coupled microstrip lines
    - Coupled striplines
    - Coplanar strips
    - Etc...

The microstrip line and the coplanar waveguide are the most used transmission lines.
Transmission lines in MMIC technology

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Transmission lines in MMIC technology

- Microstrip line:
  - Structure:
    - $W$: width of the conductor strip,
    - $t$: thickness of the conductor strip,
    - $h$: thickness of the substrate.
  - Characteristics of materials:
    - $\sigma$: conductivity of the conductor,
    - $\epsilon$: permittivity of the substrate,
    - $\text{tg}\delta$: dielectric loss tangent of the substrate.
  - EM field:
    - Quasi-TEM propagation.
Transmission lines in MMIC technology

- Microstrip line:
  
  - Characteristic impedance:
    
    \[ Z_c = \frac{Z_{\text{vacuum}}}{2\pi \sqrt{\varepsilon_{\text{eff}}}} \ln \left( C \frac{h}{W} + \sqrt{1 + \left( \frac{2h}{W} \right)^2} \right) \]
    
  - Effective permittivity:
    
    \[ \varepsilon_{\text{eff}} = \frac{\varepsilon_r + 1}{2} + \frac{\varepsilon_r - 1}{2} \left( 1 + 10 \frac{h}{W} \right)^{-ab} \]
    
    \[ C = 6 + (2\pi - 6) \exp \left( -\left( 30.666 \frac{h}{W} \right)^{0.7528} \right) \]
    
    \[ a = 1 + \frac{1}{49} \ln \left( \frac{u^4 + \left( \frac{u}{52} \right)^2}{u^4 + 0.432} \right) + \frac{1}{18.7} \ln \left( 1 + \left( \frac{u}{18.1} \right)^3 \right) \]
    
    \[ b = 0.564 \left( \frac{\varepsilon_r - 0.9}{\varepsilon_r + 3} \right)^{0.053} \]
    
    \[ u = \frac{W}{h} \]
    
    \[ Z_{\text{vacuum}} = \sqrt{\frac{\mu_0}{\varepsilon_0}} = 120\pi \]
    
    \[ \mu_0 = 4\pi \times 10^{-7} \text{ (H/m)} \]
    
    \[ \varepsilon_0 = 8.842 \times 10^{-12} \text{ (F/m)} \]

Transmission lines in MMIC technology

- **Microstrip line:**
  - Example: \( \varepsilon_r = 10 \) (alumina), \( W = 5 \, \mu m \) to 1.54 mm.

![Graphs showing characteristic impedance and effective permittivity of microstrip lines vs. width](image)

For \( Z_c \approx 50 \, \Omega \Rightarrow W \approx h \)

\[
W = h = 635 \, \mu m \\
\text{or} \\
W = h = 1.54 \, mm
\]
Transmission lines in MMIC technology

- Microstrip line:
  - Total attenuation constant: $\alpha_t$
    \[ \alpha_t = \alpha_d + \alpha_c \]
  - Attenuation due to the dielectric losses of the substrate: $\alpha_d$
    \[ \alpha_d = \frac{\varepsilon_r \varepsilon_{\text{reff}} - 1}{\sqrt{\varepsilon_{\text{reff}} \varepsilon_r - 1 \lambda_0}} \tan \delta_s \]
  - Attenuation due to the losses in the conductors: $\alpha_c$
    \[ \alpha_c = \frac{R_s}{Z_c \omega W} \cdot K_r \cdot K_i \]
    \[ R_s = (\delta \sigma)^{-1} \]
    \[ \delta = \frac{2}{\omega \mu_0 \mu_c \sigma} \]
    \[ K_i = \exp \left(-1.2 \left( \frac{Z_c}{Z_{\text{vacuum}}} \right)^{0.7} \right) \]
    \[ K_r = 1 + \frac{2}{\pi} \arctan \left(1.4 \left( \frac{\Delta}{\delta} \right)^2 \right) \]

  - $R_s$ is the sheet resistance. $\delta$ is the skin depth. $\sigma$ is the conductivity of the strip.
  - $K_i$ is the current distribution factor.
  - $K_r$ is a correction term due surface roughness. $\Delta$ is the effective (rms) surface roughness of the substrate.
Transmission lines in MMIC technology

- Microstrip line:
  - Attenuation constant:
    - Example:
      - $\varepsilon_r = 10$
      - $\tan\delta_e = 0.001$
      - $\sigma = 45 \text{ MS/m}$
      - $W = 50 \mu\text{m}$
      - $h = 635 \mu\text{m}$
  
  $\Rightarrow$ The conductor losses are the preponderant factor of losses for microstrip lines.
Transmission lines in MMIC technology

- Microstrip line:
  - Operating frequency:
    - In order to prevent higher-order transmission modes, the thickness of the microstrip substrate should be limited to 10% of the wavelength.

![Graph showing maximum recommended operating frequency (GHz) vs. substrate thickness (mils) for GaAs, Alumina, and Quartz materials.](image)
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Transmission lines in MMIC technology

- **Coplanar waveguide:**
  - **Structure:**
    - $W$: width of the conductor strip,
    - $S$: width of slots between central conductor and ground planes,
    - $W_g$: width of the ground planes,
    - $t$: thickness of the conductor strip,
    - $h$: thickness of the substrate.
  - **Characteristics of materials:**
    - $\sigma$: conductivity of the conductor,
    - $\varepsilon$: permittivity of the substrate,
    - $\text{tg}\delta$: dielectric loss tangent of the substrate.
  - **EM field:**
    - Quasi-TEM propagation.
Transmission lines in MMIC technology

- Coplanar waveguide:
  - Characteristic impedance:
    \[
    Z_c = \frac{Z_{\text{vacuum}} K'(k_1)}{4 \sqrt{\varepsilon_{\text{eff}}} K(k_1)}
    \]
  - Effective permittivity:
    \[
    \varepsilon_{\text{eff}} = 1 + \frac{\varepsilon_r - 1}{2} \frac{K(k_2)K'(k_1)}{K'(k_2)K(k_1)}
    \]

\[\text{For } 0 \leq k \leq 0.707\]

\[\text{For } 0.707 \leq k \leq 1\]

- \(K: \) elliptic integral

\[
k_1 = \frac{W}{d} \quad k_2 = \frac{\sinh\left(\frac{\pi W}{4d}\right)}{\sinh\left(\frac{\pi d}{4h}\right)} \quad d = W + 2S
\]

\[
k' = \sqrt{1 - k^2}
\]


Transmission lines in MMIC technology

- **Coplanar waveguide:**
  - Example: $h = 635 \, \mu\text{m}$, $\varepsilon_r = 10$ (alumina), $S = 5$ to $100 \, \mu\text{m}$, $W = 5 \, \mu\text{m}$ to $200 \, \mu\text{m}$.

Several values of $(W,S)$ for $Z_c = 50 \, \Omega$

$\varepsilon_{\text{eff}} \approx (\varepsilon_r + 1)/2 = 5.5$
Transmission lines in MMIC technology

- Coplanar waveguide:
  - Propagation constant:
    - Different origins of losses:
      - Losses due to the dielectric substrate: $\alpha_d$
      - Losses due to the conductors: $\alpha_c$
    - Two additional phenomena at high frequencies:
      - Dispersion: $\varepsilon_{\text{eff}} = f(f)$
      - Losses by radiation: $\alpha_r$

$\Rightarrow$ Total attenuation $\alpha_t = \alpha_d + \alpha_c + \alpha_r$
Transmission lines in MMIC technology

- **Coplanar waveguide:**
  - **Phase constant:**
    \[
    \beta = \frac{2\pi}{\lambda_0} = \frac{2\pi}{\lambda_0} \sqrt{\varepsilon_{\text{eff}}} \quad \text{(rad/m)}
    \]
    \[
    \lambda_0 = \frac{c}{f} = \frac{1}{f \sqrt{\varepsilon_0 \mu_0}} = 3.10^8 \quad \text{(m)}
    \]
  - **Example:**
    - \( W = 20 \ \mu\text{m} \)
    - \( S = 10 \ \mu\text{m} \)
    - \( h = 635 \ \mu\text{m} \)
    - \( \varepsilon_r = 10 \)
    - \( \varepsilon_{\text{eff}} = 5.5 \)
    - \( Z_c = 51 \ \Omega \)
Transmission lines in MMIC technology

- Coplanar waveguide:
  - Dispersion of the effective permittivity:
    \[ \beta(f) = \frac{2\pi}{\lambda_g} = \frac{2\pi}{\lambda_0} \sqrt{\varepsilon_{\text{eff}}(f)} \text{ (rad / m)} \]
    \[ \varepsilon_{\text{eff}}(f) = \left[ \sqrt{\varepsilon_{\text{eff}}} + \sqrt{\varepsilon_r - \varepsilon_{\text{eff}}} \right]^2 \]
    \[ f_{\text{TE}} = \frac{c_0}{4h\sqrt{\varepsilon_r - 1}} \]
    \[ x = \ln \left( \frac{W}{h} \right) \]
    \[ u = 0.54 - 0.64x + 0.015x^2 \]
    \[ v = 0.43 - 0.86x + 0.540x^2 \]
    \[ b = 1.8 \]

- \( f_{\text{TE}} \) is the cutoff frequency of the lowest TE mode.
- \( \varepsilon_{\text{eff}} \) is the initial effective permittivity calculated from the previous expressions.
- \( \varepsilon_r \) is the relative permittivity of the substrate.
- \( h \) is the height of the substrate.

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- Coplanar waveguide:
  - Dispersion of the effective permittivity:

Example:
- \( W = 70 \, \mu m \)
- \( S = 40 \, \mu m \)
- \( h = 635 \, \mu m \)
- \( \varepsilon_r = 10 \)

\[ \Rightarrow \text{Very high frequency phenomena: } f > 200 \, \text{GHz} \]

\[ \Rightarrow \text{Not so high frequency phenomena if } W \approx h \]
Transmission lines in MMIC technology

- **Coplanar waveguide:**
  - **Attenuation constant:**
    - **Losses due to the dielectric substrate:** $\alpha_d$
      \[
      \alpha_d = \frac{\pi}{\lambda_0} \frac{\varepsilon_r}{\sqrt{\varepsilon_{\text{eff}}}} q \tan \delta_e \text{(Nepers/m)} = 8.686 \frac{\pi}{\lambda_0} \frac{\varepsilon_r}{\sqrt{\varepsilon_{\text{eff}}}} q \tan \delta_e \text{(dB/m)}
      \]
      \[
      q = \frac{1}{2} \frac{K(k_2)}{K'(k_2)} \frac{K'(k_1)}{K(k_1)}
      \]
      - $\varepsilon_r$ is the relative permittivity of the dielectric substrate.
      - $\tan\delta_e$ is the dielectric loss tangent of the dielectric substrate.
      - $\varepsilon_{\text{eff}}$ is the effective permittivity of the transmission line calculated from the previous expressions.
      - The attenuation constant is expressed in Nepers/m. To be expressed in dB, one can multiply the attenuation constant by 8.686 (1 Np = 8.686 dB).

Transmission lines in MMIC technology

- Coplanar waveguide:
  - Attenuation constant:
    
    \[ \alpha_c = \frac{R_c + R_g}{2Z_C} \text{(Nepers/m)} = 8.686 \frac{R_c + R_g}{2Z_C} \text{(dB/m)} \]

  - Losses due to the conductors: \( \alpha_C \)

    \[ \alpha_C = \frac{R_c + R_g}{2Z_C} \]

  - R linked to the central conductor:
    \[ R_c = \frac{R_g}{4W(1-k_i^2)K_2(k_i)} \left[ \pi + \ln \left( \frac{4\pi W}{t} \right) - k_i \ln \left( \frac{1 + k_i}{1 - k_i} \right) \right] \]

  - R linked to the ground planes:
    \[ R_g = \frac{R_g}{4W(1-k_i^2)K_2(k_i)} \left[ \pi + \ln \left( \frac{4\pi(W + 2S)}{t} \right) - \frac{1}{k_i} \ln \left( \frac{1 + k_i}{1 - k_i} \right) \right] \]

  - Sheet resistance: \( R_s = (\delta \sigma)^{-1} \) and skin effect: \( \delta = \frac{2}{\omega \mu_0 \mu_c \sigma} \)
Transmission lines in MMIC technology

- **Coplanar waveguide:**
  - **Attenuation constant:**
    - Losses due to the radiation: $\alpha_r$
      $$\alpha_r = \left(\frac{\pi}{2}\right)^5 \cdot 2 \cdot \left(1 - \frac{\varepsilon_{\text{eff}}(f)}{\varepsilon_r}\right)^2 \left(W + 2S\right)^2 \frac{\varepsilon_r^2}{c_0^3 K(k_0') K(k_0)} f^3$$
      $$k_0 = \frac{W}{W + 2S}$$
      $$k_0' = \sqrt{1 - k_0^2}$$
    - $c_0 = 3.10^8$ m/s (light velocity)
    - $K$ is an elliptic function.
    - $\varepsilon_{\text{eff}}(f)$ can be evaluated from the previous expressions of the dispersion.
    - $\varepsilon_r$ is the relative permittivity of the substrate.
    - $W$ and $S$ are the width of the central strip and of the slot of the CPW, respectively.

Transmission lines in MMIC technology

- Coplanar waveguide:
  - Total attenuation constant: $\alpha_t$

 resett 

\[ \text{Example:} \]
- \( W = 40 \, \mu\text{m} \)
- \( S = 20 \, \mu\text{m} \)
- \( h = 635 \, \mu\text{m} \)
- \( \varepsilon_r = 10 \)
- \( \tan\delta_e = 0.0001 \)
- \( t = 4 \, \mu\text{m} \)
- \( \sigma = 45 \, \text{MS/m} \)

⇒ The losses due to the conductors are the main factor of losses in planar transmission lines!
Transmission lines in MMIC technology

- Propagation in transmission lines
- Topologies of transmission lines
- Planar transmission lines
  - Microstrip line
  - Coplanar waveguide
- Coaxial cable
- Waveguides
Transmission lines in MMIC technology

- Coaxial cable:
  - Features:
    - Widely used for high frequency measurements.
    - Ultra-large bandwidth.
      - The manufacturer guarantees that the coaxial cable correctly operates up to a certain frequency.
    - Extremely low attenuation, depending on the dielectric material.
    - Various topologies:
      - choice depending on the frequency,
      - connections,
      - rigid or flexible cable…
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- Coaxial cable:
  - Structure:
  - TEM propagation:

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- Coaxial cable:
  - a is the inner radius and b the outer radius.
  - Characteristic impedance:
    \[ Z_c = \frac{60 \ln\left(\frac{b}{a}\right)}{\sqrt{\varepsilon}} \]
  - From the transmission line theory, one can obtain the total attenuation:
    \[ \alpha_t = \frac{R}{2Z_c} + \frac{GZ_c}{2} = \alpha_c + \alpha_d \]
  - Attenuation due to the dielectric material:
    \[ \alpha_c (dB/m) = \frac{3.03 \times 10^{-9} \sqrt{\varepsilon_r} \left(1 + \frac{b}{a}\right) \sqrt{f}}{b \ln(b/a)} \]
  - Attenuation due to the conductors:
    \[ \alpha_d (dB/m) = 90.94 \times 10^{-9} \sqrt{\varepsilon_r} \tan\delta_e f \]
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- Coaxial cable:
  - RLCG elements:
    
    \[ L(H / m) = \frac{\mu_0}{2\pi} \ln(b / a) \]
    
    \[ C(F / m) = \frac{2\pi\varepsilon_0\varepsilon_r}{\ln(b / a)} \]
    
    \[ R(\Omega / m) = \frac{a + b}{2\pi ab\delta\sigma} \]
    
    \[ G(S / m) = \omega C \tan \delta_e \]

- \( \delta \) is the skin depth:
  
  \[ \delta = \frac{1}{\sqrt{\pi\mu_0\sigma}} \]
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- **Coaxial cable:**
  - **Cut-off frequency:**
    - One can use a coaxial cable until the apparition of the first higher order mode, the TE\(_{01}\) propagation mode.
    - The TE\(_{01}\) cut-off frequency is:
      \[
      f_c = \frac{1}{\pi \left(\frac{a+b}{2}\right) \sqrt{\mu \varepsilon}}
      \]
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- Waveguides:
  - Features:
    - Propagation into hollow (empty) waveguide.
    - Rectangular or circular section.
    - $TE_{mn}$ or $TM_{mn}$ mode of propagation:

$\Rightarrow$ Theory of transmission lines!
Transmission lines in MMIC technology

- Waveguides:
  - Features:
    - Extremely high frequency.
    - High power applications.
    - Advantage of waveguides: not disturbed by the EM environment.
    - Even if a waveguide operates as a high-pass filter, the manufacturer guarantees that the waveguide correctly operates for a certain bandwidth because of disturbing higher frequency modes.
    - The bandwidth is directly linked to the size of the waveguide.
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- Waveguides:
  - Conventional rectangular waveguides:

<table>
<thead>
<tr>
<th>Waveguide type</th>
<th>Frequency range (GHz)</th>
<th>Width a (mm)</th>
<th>Height b (mm)</th>
</tr>
</thead>
<tbody>
<tr>
<td>WR650</td>
<td>1.12 – 1.70</td>
<td>165.10</td>
<td>82.55</td>
</tr>
<tr>
<td>WR510</td>
<td>1.45 – 2.20</td>
<td>129.54</td>
<td>64.77</td>
</tr>
<tr>
<td>WR430</td>
<td>1.70 – 2.60</td>
<td>109.22</td>
<td>54.61</td>
</tr>
<tr>
<td>WR340</td>
<td>2.20 – 3.30</td>
<td>86.36</td>
<td>43.18</td>
</tr>
<tr>
<td>...</td>
<td>...</td>
<td>...</td>
<td>...</td>
</tr>
<tr>
<td>WR18</td>
<td>40.0 – 60.0</td>
<td>4.775</td>
<td>2.388</td>
</tr>
<tr>
<td>WR14</td>
<td>50.0 – 75.0</td>
<td>3.759</td>
<td>1.880</td>
</tr>
<tr>
<td>WR12</td>
<td>60.0 – 90.0</td>
<td>3.0948</td>
<td>1.5494</td>
</tr>
<tr>
<td>WR10</td>
<td>75.0 – 110.0</td>
<td>2.5400</td>
<td>1.2700</td>
</tr>
</tbody>
</table>
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- Waveguides:
  - Propagation modes: $\text{TE}_{mn}$ or $\text{TM}_{mn}$
    - $m$ is the number of $\frac{1}{2}$ wavelength variations of field in the “a” direction.
    - $n$ is the number of $\frac{1}{2}$ wavelength variations of field in the “b” direction.

- Rectangular waveguide:
- Circular waveguide:

![Rectangular Waveguide](image1.png)

$\text{TE}_{10}$ mode

![Circular Waveguide](image2.png)

$\text{TE}_{11}$ mode
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- Waveguides:
  - Rectangular waveguide: TE_{10} mode

- The field expressions are normalized respect to the maximal value of \( E_y \) which is obtained at the center of the waveguide at \( x = a/2 \):

\[
\begin{align*}
\overline{E}_0 &= -j\overline{E} \frac{\lambda_c}{\lambda} \\
\overline{H}_0 &= \overline{E}_0 \sqrt{\frac{\varepsilon}{\mu}}
\end{align*}
\]

- The fields of the fundamental mode are:

\[
\begin{align*}
\overline{E}_y &= \overline{E}_0 \sin \frac{\pi x}{a} \exp \left( -j \frac{2 \pi z}{\lambda_g} \right) \\
\overline{H}_x &= -\overline{H}_0 \frac{\lambda}{\lambda_g} \sin \frac{\pi x}{a} \exp \left( -j \frac{2 \pi z}{\lambda_g} \right) \\
\overline{H}_z &= j\overline{H}_0 \frac{\lambda}{\lambda_c} \cos \frac{\pi x}{a} \exp \left( -j \frac{2 \pi z}{\lambda_g} \right)
\end{align*}
\]

\( \lambda_c = 2a \)

\( f_c = \frac{c_0}{\lambda_c} = \frac{c_0}{2a} \)

\( \lambda_g = \lambda \left[ 1 - \left( \frac{\lambda^2}{4a^2} \right) \right]^{-1/2} \)
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- **Waveguides:**
  - Rectangular waveguide:
    - Attenuation in rectangular waveguide:
      \[
      \alpha_i (\text{Nepers/m}) = \frac{1}{2} \frac{P_w}{P_d} = \frac{1}{a^{3/2}} \sqrt{2\pi \varepsilon \rho} \left( \frac{f_c}{f} \right)^{3/2} + \left( \frac{a}{2b} \right) \frac{f}{f_c} \sqrt{1 - \left( \frac{f_c}{f} \right)^2} \]
      - \(P_w\) is the power lost in the walls of the waveguide.
      - \(P_d\) is the power transmitted by the dielectric material.
      - \(\varepsilon\) is the permittivity of the material filling the waveguide.
      - \(c\) is the speed in the material filling the waveguide.
      - \(f_c\) is the cutoff frequency of the waveguide.
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- Waveguides:
  - Circular waveguide: TE modes (fundamental = TE_{11} mode)

- The fields of the TE modes are:

\[
\overline{E}_\rho = -\bar{H} \sqrt{\frac{\mu}{\varepsilon}} \frac{\lambda_c}{\lambda} n J_n(u)
\]

\[
\overline{E}_\phi = j\bar{H} \sqrt{\frac{\mu}{\varepsilon}} \frac{\lambda_c}{\lambda} J_n'(u)
\]

\[
\overline{H}_\rho = -j\bar{H} \frac{\lambda_c}{\lambda_g} J_n'(u)
\]

\[
\overline{H}_\phi = -\bar{H} \frac{\lambda_c}{\lambda_g} n J_n(u)
\]

\[u = k_c \rho\]

\[k_c = \frac{2\pi}{\lambda_c} \quad \lambda_c = \frac{2\pi a}{u'_{mn}}\]

- The value of \( H \) is fixed by measuring the power transmitted by the waveguide.
- \( a \) is the radius of the circular waveguide.
- \( J_n \) is the Bessel’s function of first kind and at the order \( n \).

<table>
<thead>
<tr>
<th>Roots and their derivatives of the Bessel’s functions</th>
<th>( u_{mn} )</th>
<th>( u'_{mn} )</th>
</tr>
</thead>
<tbody>
<tr>
<td>( U_{0,1} = 2.405 )</td>
<td></td>
<td>( U'_{1,1} = 1.841 )</td>
</tr>
<tr>
<td>( U_{1,1} = 3.832 )</td>
<td></td>
<td>( U'_{2,1} = 3.054 )</td>
</tr>
<tr>
<td>( U_{2,1} = 5.136 )</td>
<td></td>
<td>( U'_{0,1} = 3.831 )</td>
</tr>
</tbody>
</table>
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- Waveguides:
  - Circular waveguide:
    - Attenuation in a circular waveguide for the TE_{11} mode:
      (fundamental mode)

\[
\alpha_i(\text{Nepers/m}) = \frac{5.5 \times 10^{-5}}{a^{3/2}} \left( \frac{f}{f_c} \right)^{-1/2} + \frac{1}{2.38} \left( \frac{f}{f_c} \right)^{3/2} \sqrt{\frac{f}{f_c}} - 1
\]

- \( f_c \) is the cutoff frequency of the circular waveguide.
- \( a \) is the radius of the circular waveguide.
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Thank you for your attention!