Esame Statistica: 22/05/2017

A] a) $P(I) + P(C) = 1$, $P(C) = 2P(I) \Rightarrow \begin{cases} P(I) + 2P(I) = 1 \Rightarrow P(I) = \frac{1}{3} \\ \frac{1}{3} + P(C) = 1 \Rightarrow P(C) = \frac{2}{3} \end{cases}$

b) ~~[illegible]~~ • $P(I) \cdot P(x|I) = \frac{1}{3} 0{,}95 = 0{,}31667$

• $P(C) \cdot P(x|C) = \frac{2}{3} 0.1 = 0{,}06667$

Dunque $P(I|x) = \dfrac{0{,}31667}{0{,}31667 + 0{,}06667} = 82{,}608\%$

$P(C|x) = \dfrac{0{,}0667}{0{,}31667 + 0{,}06667} = 17{,}392\%$

c) • $P(I) \cdot P(y|I) = \frac{1}{3} 0{,}05 = 0{,}01667$

• $P(C)\, P(y|C) = \frac{2}{3} 0{,}9 = 0{,}6$

Dunque $P(I|x) = \dfrac{0{,}01667}{0{,}01667 + 0{,}6} = 2{,}7032\%$

$P(C|x) = \dfrac{0{,}6}{0{,}01667 + 0{,}6} = 97{,}2968\%$

B] a) $\mathbb{E}(R) = 0 \cdot 0{,}6 + 100.000 \cdot 0{,}4 = 40.000$

b) $\mathbb{E}(R) - f = 40000 - 37000 = 3000$

c) Conviene se $U(f) > \mathbb{E}[U(R)]$

Dunque • $U(f) = -e^{-\frac{37000}{100.000}} = -e^{-0{,}37} \cong -0{,}6907$

• $\mathbb{E}(U(R)) = -e^{-\frac{0}{100.000}} 0{,}6 - e^{-\frac{100.000}{100.000}} 0{,}4 = -1 \cdot 0{,}06 - e^{-1} \cdot 0{,}4 \cong -0{,}7472$

$\Rightarrow$ Conviene accettare $\mathbb{E}(U(R)) < U(f)$

C] a) $t\int_0^2 (4x - 2x^2)\,dx = t\left[2x^2 - \frac{2x^3}{3}\right]\Bigg|_{x=0}^{x=2} = 1$

b) $t = \frac{3}{8}$

c) $P\{X > 1\} = \int_1^{\infty} f(x)\,dx = \frac{3}{8}\int_1^2 (4x - 2x^2)\,dx = \frac{1}{2}$

D] $H_0: \mu = 5$; $H_1: \mu < 5$; $-z_\alpha = -1{,}645$ $\quad n = 16$

SI RIFIUTA H_0 SE $\frac{\bar{x} - \mu_0}{\sigma/\sqrt{n}} < -1{,}645$

SE $\bar{x} < \bar{x}_c = \mu_0 - 1{,}645\,\frac{\sigma}{\sqrt{n}} = 4.959$

$\Rightarrow \frac{4962 - 5}{0.1/\sqrt{16}} = -1{,}52 \Rightarrow -1{,}52 > -1{,}645$

$\bar{x} = 4962 > \bar{x}_c = 4959$ $\quad\Rightarrow$ H_0 È RIFIUTATO

- P-VALUE $\Rightarrow P(Z \leq -1{,}52) = 0{,}0643$

$\hookrightarrow$ $P(Z \leq 1{,}52) = 1 - 0{,}9357$

Per cui $0{,}0643 > \alpha = 0{,}05$ che corrobora la conclusione