REGIONAL ANALYSIS OF RAINFALL-DEPTH-DURATION EQUATION FOR SOUTH ITALY

By Vito Ferro1 and Paolo Porto2

ABSTRACT: The two-component extreme value (TCEV) distribution, which is currently applied in Italy for the regional rainfall and flood frequency analysis, and its hierarchical parameter estimating procedure, are briefly reviewed. Then, a new rainfall-depth-duration equation, based on TCEV distribution and applicable for the hypothesis of both duration-dependent and duration-independent parameter estimates, is proposed. Finally, for the investigated Italian geographical regions (Apulia, Basilicata, Calabria, and Campania), the regionalization procedure is developed, and the rainfall-duration-frequency factor appearing in the rainfall-depth-duration equation is calibrated. The isoline maps of the mean rainfall having a duration equal to 24 h and of the exponent n, appearing in the power equation linking the mean μ, of the TCEV distribution with the duration t, are also plotted.

INTRODUCTION

For designing hydraulic structures or for evaluating the effectiveness of a natural or manmade drainage system, a "design storm" has to be provided. According to Chow et al. (1958), a design storm is a precipitation pattern used by a hydrologic model to determine storm water runoff.

Most of the methods for constructing synthetic design storms are linked to the use of a rainfall-depth-duration (RDD) equation (Bedient and Huber 1992); in other words, the knowledge of the rainfall depth h_t occurring in a period of t hours and having a return period of T years is necessary. The RDD equation is established by point precipitation data; therefore, a duration-area-rainfall depth relationship must also be introduced for studying large areas.

At first, for estimating h_t, by using a sample of the annual maximum values of the rainfall depth h of given duration t, a choice of a theoretical cumulative distribution function (CDF) and an estimation of its parameters are necessary. Any theoretical procedure for deciding which distribution must be used in a rainfall frequency analysis is not yet available (Alexander et al. 1969; Cunnane 1985). In the past, the procedure usually followed was to check and compare the suitability of several candidate distributions. The choice is based on an a posteriori examination of the goodness-of-fit of the theoretical CDF to the empirical one.

This procedure has a high uncertainty if the size of the at-site sample is small and it can lead to inconsistent results, i.e., different theoretical CDFs can be fitted to sample series of neighboring recording rain gauges (Versace et al. 1989).

According to Cunnane (1985), the statistical characteristics (kurtosis, skewness coefficient, coefficient of variation) obtained from single hydrological records of the usual length can have relatively large standard errors as well as being biased downward. As a consequence, the attempt to infer the exact form of the probability distribution for maximum annual rainfall from statistical characteristics of a single series would most likely be erroneous.

Experience shows that when the tail of the distribution

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of interest, as for extreme rainfall and flood analysis, quantiles estimated with different theoretical models can differ substantially, even if each theoretical CDF fits the data well in the central area (for T < 10 years) (Landwehr et al. 1978, 1980).

For this reason, some authors (Kuczera 1982) proposed using a probability distribution and a method of parameter estimation, which are robust with respect to quantile estimation in the selected probability range over a reasonable set of plausible distributions.

Some other studies (Wallis et al. 1974) focused attention on the higher moments of distribution because of the recognized influence of these moments on the shape of the tail. Hydrologists have often used the sample skewness coefficient G as a key index about the form of the rainfall and flood frequency distribution.

In particular, the skewness coefficient will be estimated by the following relationship, which is to be used for samples with a short sample size N:

$$G = \frac{N}{(N-1)(N-2)} m_3 \frac{1}{m_2^2}$$

where $m_3$ = second-order moment; and $m_2$ = third-order moment.

The study of Matalas et al. (1975) suggested that an important characteristic of the maximum annual data that should be taken into account when selecting a distribution is the condition of the separation in the skewness. Matalas et al. (1975), examining 1,351 stream gauging stations in 14 hydrologically homogeneous regions of the United States, found that the skewness of 10-, 20-, and 30-year nonoverlapping flood series was, for each of the 14 regions, more variable than that calculated from similar-sized data sets randomly generated from a group of commonly used flood frequency distributions (normal, lognormal, Gumbel, Pearson Type 3, Weibull, Pareto, and uniform). The separation effect observed by Matalas et al. (1975) allowed the establishment of the criterion that a theoretical probability distribution is adequate to rainfall and flood frequency analysis if it is able to reproduce at least as much variability in flood or rainfall characteristics (particularly skewness coefficient) as observed in empirical data sets. Subsequent experiments showed that neither log-Pearson Type 3 distribution (Landwehr et al. 1978) nor the general extreme value distribution (Cunnane 1985) could account for this variability.

Two distributions, which are able to produce data having a skewness coefficient sufficiently variable to account for the separation effect, are the Wakeby distribution (Houghton 1978) and the two-component extreme value distribution (TCEV) (Rossi et al. 1984).

Matalas et al. (1975) showed that the use of small samples
for estimating $G$ and autocorrelation cannot explain the separation phenomenon. On the other hand, Wallis et al. (1977) examined the spatial mixing and nonstationarity of skewness and concluded that the condition of separation may be explained by one of these factors.

Presently, the notion of separation of skewness is used as a criterion for justifying the use of one type of probability distribution for rainfall and flood frequency analysis; this choice is coupled with a regional procedure for estimating the parameters.

For reducing the uncertainty of the estimated parameters of the chosen theoretical distribution, for improving the at-site quantile estimate $h_{1,T}$ based on limited data (particularly when the return period $T$ is much higher than the sample size $N$), and to estimate $h_{1,T}$ for ungaged sites, a regional estimation technique is necessary (Wiltshire 1986; Cunnane 1988).

Regionalization is a tool to extend the length of the historical samples and to reduce time sampling errors. Regionalization introduces errors due to space disturbance (Matalas and Gilroy 1968), and interstation correlation (Stedinger 1983) leads to estimates of the hydrological variable that are less accurate than they would be if the samples were independent.

The time sampling variability increases with the order $m$ of the moments, whereas the ratio between the space disturbance and the time sampling variability decreases with $m$ (Fiorentino et al. 1987; Rossi and Villani 1992). Stedinger (1983) showed that the influence of the interstation correlation decreases with $m$.

Regionalization should always be used in statistical analysis of extreme hydrological events because of the large influence that higher moments (skewness coefficient) exert on the shape of the tail of the distribution (Wallis et al. 1974).

In conclusion, the choice of a theoretical CDF should be made, taking into account the following criteria:

1. Information about the physical phenomena (rainfall, flood) is useful to establish the structure of the model (theoretical basis).
2. The theoretical CDF has to be able to reproduce the statistical characteristics (skewness coefficient, coefficient of variation, mean) of the observed samples (descriptive ability).
3. The model parameters must have physical meaning to make the regionalization easy (physical basis).
4. The $T$-year variable estimate $x_T$ must be efficient (minimum mean square error) and robust (resistant to departures from the hypothesis) (predictive ability).

According to these criteria, in recent years, the Italian National Research Group for the Prevention of Hydro-Geological Disasters developed a national research project, VAlutazione Piane Italia (VAPI), which aimed at unifying the analysis of the annual rainfall series and annual flood series measured in Italy (Rossi and Villani 1992). In comparison with the “flood studies report” carried out by the English Natural Environmental Research Council and the “guidelines for determining flood frequency” developed in the United States, the VAPI project also links the preliminary statistical analysis of the daily and hourly rainfalls to the regional analysis of flood frequency and to the estimate of the index flood [Consiglio Nazionale delle Ricerche (CNR 1994)].

The VAPI project is based on the use of the TCEV distribution (Rossi et al. 1984) and its hierarchical regionalization procedure (Fiorentino et al. 1987; Versace et al. 1989).

The estimate of the four parameters of the TCEV distribution was carried out in the different Italian regions, taking into account the hypothesis that the parameters are both duration dependent (DD) and duration independent (DI).

![FIG. 1. Comparison between Empirical Skewness CDF of 19 Calabrian Rain Gauges Corresponding to 1 h and Daily Rainfall Historical Sequences (after Fiorentino et al. 1984)](image)

### TABLE 1. Characteristic Data of Investigated Recording and Nonrecording Rain Gauges

<table>
<thead>
<tr>
<th>Region</th>
<th>$n_r$ (2)</th>
<th>$N_r$ (3)</th>
<th>$n_{nr}$ (4)</th>
<th>$N_{nr}$ (5)</th>
<th>$SP_r$ (6)</th>
<th>$SP_{nr}$ (7)</th>
</tr>
</thead>
<tbody>
<tr>
<td>Basilicata</td>
<td>49</td>
<td>30</td>
<td>68</td>
<td>48</td>
<td>1921–1987</td>
<td>1921–1985</td>
</tr>
<tr>
<td>Calabria</td>
<td>85</td>
<td>31</td>
<td>53</td>
<td>49</td>
<td>1921–1987</td>
<td>1921–1972</td>
</tr>
</tbody>
</table>
In this paper, after a brief review of the TCEV distribution and the hierarchical regionalization estimating procedure of its parameters, a new RDD relationship, applicable for both DI and DD cases, is proposed. The rainfall-duration frequency factor appearing in the RDD relationship is calibrated for four Italian geographical regions (Apulia, Basilicata, Calabria, and Campania).

Finally, for each region, isoline maps of the mean rainfall \( \mu_{24} \), having a duration \( t = 24 \text{ h} \), and of the exponent \( n \), appearing in the power equation linking the mean \( \mu \) of the TCEV distribution with the duration \( t \), are plotted. These maps are useful for estimating \( \mu_{24} \) and \( n \) at ungauged sites.

**REVIEW OF TCEV DISTRIBUTION AND HIERARCHICAL REGIONALIZATION TECHNIQUE**

The TCEV distribution, as proposed by Rossi et al. (1984), was developed from the observation of the historical flood sequences measured in Italy and was also verified using maximum rainfall data measured in different Italian regions (Rossi and Villani 1992) and United Kingdom flood data (Arnell and Gabriele 1988). The authors hypothesized that rainfalls or floods could be described by two probability distributions, and the annual maximum value could be considered as the larger of two sets of independent and identically distributed random variables. The TCEV distribution was derived by assuming that both processes had rates of occurrence defined by the Poisson distribution and exponentially distributed magnitudes (Arnell and Gabriele 1988).

In other words, the TCEV distribution takes into account that one or more annual maximum values of the observed hydrological variable (rainfall depth \( h \), occurring in a period of \( t \) hours) may be much higher than the bulk of the remaining data. In each historical sample of \( h \), two components are distinguishable: (1) The basic component, which takes into account the usual values; and (2) the outlying component, which takes into account the extreme values. This choice is justified if maximum rainfalls and floods are due to storms with different meteorological characteristics (Rossi and Villani 1992). The CDF of the TCEV distribution \( F(h) \) is a probabilistic model with four parameters and is equal to the product of two extreme value Type I (EV1) distributions (an EV1 distribution for each component)

\[
F(h) = \exp \left[ -\lambda_1 \exp \left( \frac{h}{\theta_1} \right) - \lambda_2 \exp \left( \frac{h}{\theta_2} \right) \right]
\]

in which \( \lambda_1, \lambda_2 = \text{shape parameters}; \) and \( \theta_1, \theta_2 = \text{scale parameters}, \) respectively, of the basic (component 1) and outlying (component 2) component.

**TABLE 2. Regional Parameters Estimated by Daily Rainfall Data and Rainfall Depth Having \( t = 24 \text{ h} \)**

<table>
<thead>
<tr>
<th>Region</th>
<th>( \lambda'_1 )</th>
<th>( \Theta'_1 )</th>
<th>( \lambda'_2 )</th>
<th>( \Theta'_2 )</th>
</tr>
</thead>
<tbody>
<tr>
<td>Apulia</td>
<td>0.772</td>
<td>2.351</td>
<td>0.923</td>
<td>1.930</td>
</tr>
<tr>
<td>Basilicata</td>
<td>0.096</td>
<td>2.630</td>
<td>0.550</td>
<td>1.784</td>
</tr>
<tr>
<td>Calabria</td>
<td>0.418</td>
<td>2.154</td>
<td>0.450</td>
<td>2.048</td>
</tr>
<tr>
<td>Campania</td>
<td>0.360</td>
<td>2.136</td>
<td>0.412</td>
<td>1.886</td>
</tr>
</tbody>
</table>

According to the theoretical derivation of the TCEV distribution (Rossi et al. 1984), \( \lambda_i (i = 1, 2) \) is the mean number of events (maximum rainfall, flood), which belong to each component \( i \), whereas \( \theta \) represents the at-site central value of the hydrological variable. The magnitude of the parameters are related by \( \lambda_1 \gg \lambda_2 \) and \( \theta_i \ll \theta_2 \), given the goal of modeling both central tendency and extremes with the same distribution. In other words, the basic component is characterized by a high number of events and by values of the hydrological variable less than those corresponding to the outlying component.

Introducing the standardized variable \( y = (h/\theta_i) - \ln \lambda_i \), (2) becomes

\[
F(y) = \exp \left[ -\exp(-y) - \Lambda^{*} \exp \left( \frac{y}{\Theta^{*}} \right) \right]
\]

in which \( \Lambda^{*} = \lambda_i/(\lambda_i^{\text{usr}}) \) = shape parameter; and \( \Theta^{*} = \theta_i/\theta_2 = \text{scale parameter.} \)

The proportion \( p \) of data values that belong to the outlier component can be calculated by the following equation (Beran et al. 1986):

\[
p = \frac{\Lambda^{*}}{\Theta^{*}} \sum_{j=0}^{\infty} \frac{(-1)^j}{j!} \Lambda^{*j} \Gamma \left( \frac{j + 1}{\Theta^{*}} \right)
\]

in which \( \Gamma \) is the gamma function.

The moments of the TCEV distribution were derived by Beran et al. (1986), assuming that the distribution is continuous. In particular, the mean \( \mu \), of the TCEV distribution is

\[
\mu = \theta_i (\ln \lambda_i + 0.5772) - \theta_i \sum_{j=1}^{\infty} \frac{(-1)^j \Lambda^{*j}}{j!} \Gamma \left( \frac{j}{\Theta^{*}} \right)
\]

where the first term is the mean of the sequences belonging to the basic component and the second term may be considered as a correction due to the contamination of the basic series by the outlier series.

Introducing the dimensionless variable \( h' \), which is equal to the ratio between \( h \) and the mean \( \mu \) of the TCEV distribution, the following CDF of the \( h' \) variable, named the growth curve, is deduced:

![Comparison among Empirical Skewness CDF of 24-h Rainfall Historical Sequences and Two Skewness CDF of Samples Generated by Monte Carlo Technique](image-url)
According to (5), the ratio $\mu_t/\theta_t$ is dependent on $\lambda_t$, $\Lambda^*$, and $\Theta^*$; therefore, only these three parameters are necessary to define the growth curve. Eq. (6), with known values of the three parameters $\lambda_t$, $\Lambda^*$, and $\Theta^*$, allows the calculation of the values $h_{t,r}$ of the hydrologic variable $h_t'$ corresponding to a return period of $T$ years (i.e., conversion of growth curve to quantile). Taking into account the definition of the dimensionless variable $h_t'$, the rainfall depth $h_{t,r}$ is computed by the following equation:

$$h_{t,r} = h_\mu \sqrt{\frac{T}{t}}$$

Eq. (7) is the RDD relationship, in which the mean $\mu_t$ of the TCEV is assumed equal, for each recording rain gauge, to the sample mean of the rainfall $h_t$ of given duration $t$. For each recording rain gauge, $\mu_t$ depends on duration $t$ (h) according to the following relationship (Viparelli 1964; Versace et al. 1989; Cannarozzo et al. 1995):

$$\mu_t = a t^n$$

in which $a$ and $n$ = numerical constants to estimate, for each recording rain gauge, by the least-squares method.

For the Calabrian region, Fiorentino et al. (1984) compared the skewness CDF of 19 samples, having a sample size $N \geq 30$ years, for which the annual maximum daily rainfalls $h_d$ and the annual maximum values of the rainfall depth $h_1$ of duration $t = 1$ h were available in the same recording period (Fig. 1). The authors, also comparing for the daily and hourly series the mean value (1.21 and 1.09, respectively) and standard deviation (0.69 and 0.67, respectively) of the skewness, hypothesized that the two CDFs were not statistically different.

Beran et al. (1986) showed that the skewness coefficient $\gamma_t$ of the TCEV distribution assumes the following expression (Rossi and Villani 1992):

$$\gamma_t = \frac{3.7553 - 3S_2 - 3.4632S_3 + 6S_4 - 3.9350S_5 + 3.4632S_6 - S_7^0}{(1.6448 + 2S_1 + 1.1544S_1 - S_7^0)^2}$$

in which $S_i$ ($i = 0, 1, 2$) is

$$S_i = \sum_{j=1}^{\infty} \frac{(-1)^j \Lambda^* \Gamma(j/\Theta^*)}{j!}$$

and $\Gamma(j/\Theta^*)$ = $j$th derivative of the $\Gamma$ function.

Eqs. (9) and (10) show that the skewness coefficient of the TCEV distribution depends on $\Lambda^*$ and $\Theta^*$ only. According to these results (skewness coefficient CDF is DI, $\gamma_t$ depends on $\Lambda^*$ and $\Theta^*$), Fiorentino et al. (1984) assumed the hypothesis that $\Lambda^*$ and $\Theta^*$ parameters were DI for the Calabrian region.

Versace et al. (1989) developed the regional analysis of Calabrian rainfall also by using the hypothesis that the $\lambda_t$ parameter is DI. In other words, for the Calabrian region, the three

$$F(h'_t) = \exp \left\{ -\lambda_t \left[ \exp \left( \frac{\mu_t}{\theta_t} \right) \right]^\theta - \Lambda^* \lambda_t^{\theta^*} \left[ \exp \left( \frac{\mu_t}{\theta^*} \right) \right]^{\theta^*} \right\}$$

(6)

FIG. 4. Homogeneous Subregions of Basilicata and Calabria

FIG. 5. Comparison among Empirical CDF of Coefficient of Variation of 24-h Rainfalls and Theoretical Ones Generated by TCEV with Regional Parameters Estimated by $h_{24}$ or Daily Rainfalls
parameters of the growth curve were assumed DI and were estimated using the \( h_d \) samples recorded at rain gauges. At present, the analysis developed by the VAPI project in many Italian geographical regions reproduces the procedure \( (\lambda^*, \Theta^*, \lambda_i) \) and \( \lambda_i \) are DI) established for the Calabrian region.

If the DI hypothesis is used, the RDD relationship becomes

\[ h_{i,t} = h_d \mu_t, \quad (11) \]

in which \( h_{i,t} = \text{dimensionless} \ h_d \) calculated by (6), in which \( \Lambda^* = \Lambda^*_{(h)} \), \( \Theta^* = \Theta^*_{(h)} \), and \( \lambda_i = \lambda_{i,(h)} \). i.e., the parameters of TCEV distribution are estimated by \( h_d \) samples.

In contrast with these results, Cannarozzo et al. (1995) discovered that the TCEV parameters \( \Lambda^*, \Theta^*, \lambda_i \) are DD, analyzing the rainfall depth \( h_i \) of duration \( t = 1, 3, 6, 12, \) and 24 h recorded at 172 Sicilian recording rain gauges. In other words, for Sicily, the DD hypothesis does not allow one to assume, as an estimate of \( \Lambda^*, \Theta^*, \lambda_i \), the values \( \Lambda^*_{(h)}, \Theta^*_{(h)} \), \( \lambda_{i,(h)} \) obtained from the regional analysis of the daily rainfalls \( h_d \). Therefore, for the DD hypothesis, the RDD relationship is (7).

The uncertainty in estimating the TCEV parameters, particularly for the outlying component, is great when a single rainfall sample is used. For this reason a regional estimate technique of TCEV parameters was developed (Beran et al. 1984; Fiorentino et al. 1987).

The regionalization procedure is hierarchical and broken down into three sequential levels.

At the first level the procedure (Fiorentino et al. 1985, 1987) assumes that the prevailing meteorological mechanism, including the underlying conditions, determining the outlying component is similar within a homogeneous region. At site, the relationship between the basic and outlying component is constant and the parameters \( \Lambda^* \) and \( \Theta^* \) of the regional model

\[ \begin{array}{|c|c|c|c|c|c|c|} \hline \text{Region} & \text{Subregion} & \text{t} & \text{\( \mu(G) \)} & \text{\( S(G) \)} & \text{\( \lambda_i \)} & \text{\( \mu_i/\theta_i \)} \\ \hline \text{Apulia} & \text{Apulia} & 1 & 0.358 & 1.830 & 15.136 & 3.819 \\ \text{Basilicata} & \text{A} & 1–24 & 0.923 & 1.930 & 28.248 & 5.165 \\ \text{Calabria} & \text{B} & 1–24 & 0.550 & 1.784 & 26.203 & 4.592 \\ \text{Campania} & \text{J} & 1 & 0.057 & 3.131 & 14.202 & 3.383 \\ \text{Campania} & \text{C} & 1 & 0.057 & 3.131 & 14.246 & 3.386 \\ \text{Campania} & \text{T} & 1–24 & 0.450 & 2.048 & 17.798 & 4.177 \\ \text{Campania} & \text{Campania} & 1 & 0.021 & 2.017 & 16.275 & 3.404 \\ \text{Campania} & \text{Campania} & 3–24 & 0.360 & 2.136 & 39.437 & 4.867 \\ \hline \end{array} \]

\[ \text{TABLE 4. Parameters of Growth Curve of Each Investigated Region} \]

FIG. 6. Comparison between Empirical and Theoretical Growth Curve \( F(h_{i,t}) \)
TABLE 5. Comparison between Historical and Generated Values of Mean and Standard Deviation of Coefficient of Variation

<table>
<thead>
<tr>
<th>Region (1)</th>
<th>Subregion (2)</th>
<th>t (h)</th>
<th>Historical μ(CV) (4)</th>
<th>Generated M(CV) (5)</th>
<th>Historical σ(CV) (6)</th>
<th>Generated S(CV) (7)</th>
</tr>
</thead>
<tbody>
<tr>
<td>Apulia</td>
<td>Apulia</td>
<td>1</td>
<td>0.428</td>
<td>0.422</td>
<td>0.077</td>
<td>0.075</td>
</tr>
<tr>
<td>Basiliaca</td>
<td>A</td>
<td>24</td>
<td>0.380</td>
<td>0.373</td>
<td>0.068</td>
<td>0.068</td>
</tr>
<tr>
<td></td>
<td>B</td>
<td>24</td>
<td>0.414</td>
<td>0.424</td>
<td>0.093</td>
<td>0.076</td>
</tr>
<tr>
<td></td>
<td>C</td>
<td>24</td>
<td>0.308</td>
<td>0.296</td>
<td>0.061</td>
<td>0.060</td>
</tr>
<tr>
<td>Calabria</td>
<td>J</td>
<td>1</td>
<td>0.450</td>
<td>0.443</td>
<td>0.125</td>
<td>0.086</td>
</tr>
<tr>
<td></td>
<td>T</td>
<td>24</td>
<td>0.457</td>
<td>0.460</td>
<td>0.106</td>
<td>0.094</td>
</tr>
<tr>
<td>Campania</td>
<td>Campania</td>
<td>1</td>
<td>0.381</td>
<td>0.355</td>
<td>0.070</td>
<td>0.063</td>
</tr>
<tr>
<td></td>
<td>Campania</td>
<td>24</td>
<td>0.359</td>
<td>0.364</td>
<td>0.079</td>
<td>0.082</td>
</tr>
</tbody>
</table>

The rainfall data used in this research are the annual maximum rainfalls with 1-, 3-, 6-, 12-, and 24-h duration and daily rainfalls published by the Italian Hydrographic Service. For each region, Table 1 lists the number of recording rain gauges and the corresponding mean sample size N, the number N of nonrecording rain gauges and the mean sample size N of the daily rainfalls. Table 1 also lists the sampling period of the recording SP and nonrecording SP rain gauges.

STATISTICAL ANALYSIS OF RAINFALL DEPTHS WITH t = 24 h

At the first level of the hierarchical regionalization analysis, according to VAPI procedure, each geographical region (Apulia, Basilica, Calabria, and Campania) has a total area equal to 58,000 km², characterized by different climatic regimes due to the particular orography of each region (Ferro and Bagarello 1996).

FIG. 7. Comparison between Pairs (T, h) Calculated by Eqs. (6) and (15)

DATA USED IN THIS RESEARCH

The four investigated regions (Apulia, Basilica, Calabria, and Campania) having a total area equal to 58,000 km², are characterized by different climatic regimes due to the particular orography of each region (Ferro and Bagarello 1996).
lia, Basilicata, Calabria, and Campania) is considered to be a homogeneous region with regard to the skewness coefficient. To verify the spatial homogeneity hypothesis for skewness coefficient, Monte Carlo experiments have to be carried out for generating sequences having a fixed size and distributed according to TCEV of known regional parameters $\Lambda^*$ and $\Theta^*$. For all generated sequences the corresponding skewness coefficient is estimated by (1) and the regional CDF of skewness of generated samples is obtained. The region is considered homogeneous with respect to skewness coefficient if the skewness CDF of the historical samples is coincident with the skewness CDF of Monte Carlo generated samples (Fiorentino et al. 1985; Arnell and Gabriele 1988). For each geographical region, the number of Monte Carlo simulations was preliminarily established. The analysis shows that the mean value $m(G)$ and standard deviation $s(G)$ of the skewness coefficient of generated samples, having a sample size equal to $N_R$, is stable with the number of Monte Carlo simulations, and 1,000 simulations are sufficient to obtain a robust estimate.

In conclusion, the analysis showed that for Basilicata the regional parameters $\Lambda^*$ and $\Theta^*$ can be assumed to be DI, whereas for the other three regions (Apulia, Calabria, and Campania) the parameters $\Lambda^*$ and $\Theta^*$ must be separately estimated for $t = 1$ h and for a duration ranging from 3 to 24 h. As a consequence, for Basilicata, the rainfall-duration frequency factor $f(t, T)$ is always equal to 1, whereas for the other three regions $f(t, T) = 1$ for $3 \leq t \leq 24$ h.

In south Italy daily rainfalls and 24-h rainfalls are generally due to the same rainfall storms, and therefore, the empirical CDF of the skewness coefficient of the annual maximum $h_{24}$ rainfalls would be coincident with the empirical CDF of the skewness coefficient of $h_{24}$ rainfalls. In other words, the regional parameters of the $h_{24}$ rainfalls could be estimated using the hydrological information of the daily rainfall samples, which generally have a greater sample size (Table 1) (Can-narozzo et al. 1995).

In Fig. 2, as an example of Basilicata and Campania, the empirical CDF for the skewness coefficient of the rainfall depth $h$ are plotted. Fig. 2 shows that the DI hypothesis can be accepted only for Basilicata, whereas for Campania and the other two regions, the CDF of the skewness coefficient of the rainfall depth occurring in a period of 1 h is clearly distinguished from the other empirical distributions [Fig. 2(b)].
of historical samples and the estimates $M(G)$ and $S(G)$ obtained by generating the 1,000 Monte Carlo technique samples distributed according to TCEV with estimates of regional parameters equal to $\Lambda^*_L$ and $\Theta^*_L$ for Campania, and to $\Lambda^*_L$ and $\Theta^*_L$ for the remaining three regions. The good agreement between the empirical and generated mean value and the standard deviation of skewness confirms the suitability of the chosen estimate criterion of the regional parameters of $h_{24}$ rainfalls.

At the second level of the regional analysis in each subregion, the $\lambda_1$ parameter is constant and the growth curve [(6)] is completely defined. A good agreement between the empirical and theoretical growth curve $F(h_{24}^*)$, as an example for Campania and Calabria, can be seen in Fig. 6. Table 4, for each geographical region and homogeneous subregion, lists the parameters $\Lambda^*$, $\Theta^*$, $\lambda_1$, and $\mu_0/\theta_1$ of the growth curve [(6)].

Table 5 shows, for each investigated subregion, the comparison between the mean value $\mu(CV)$ and standard deviation $\sigma(CV)$ of the coefficient of variation of historical samples and the estimates $M(CV)$ and $S(CV)$ obtained by the Monte Carlo technique. The good agreement between empirical and generated values of mean and standard deviation of coefficient of variation confirms the suitability of the determined growth curves.

Because the distribution function $F(h^*)$ is implicit for $h^*$, for each subregion and $t = 24$ h, the following rough relationship is proposed:

$$h_{24}^* = a_{24} + b_{24} \ln T$$

in which $a_{24}$ and $b_{24}$ are constants to be estimated by the least-squares method. Table 6 lists, for each investigated subregion, $a_{24}$ and $b_{24}$ numerical values. Fig. 7 shows, as an example for Campania, the good agreement between the pairs $(T, h_{24}^*)$ calculated by (6) and (15).

### STATISTICAL ANALYSIS OF RAINFALL DEPTH WITH $t = 1$ h

The statistical analysis was developed for the three regions (Apulia, Calabria, and Campania) for which the DD hypothesis was recognized. Fig. 8 shows, as an example for Calabria and Campania, that TCEV with regional parameters estimated by hourly rainfall (Table 4) is able to reproduce the skewness coefficient CDF of $h$ rainfall historical sequences. The theoretical CDF of skewness coefficient is obtained by 1,000 samples, having a size equal to $N_h$ (Table 1), generated by the Monte Carlo technique. Fig. 9 shows, as an example for Calabria and Campania, the ability of the TCEV distribution, with ML estimated parameters (Table 4), to describe the empirical CDF of the coefficient of variation of generated samples. The analysis demonstrated that only for Campania [Fig. 3(a)], is the TCEV distribution with regional parameters equal to $\Lambda^*_L$ and $\Theta^*_L$ able to reproduce the empirical CDF of the skewness coefficient better than a TCEV with parameter estimates equal to $\Lambda^*_L$ and $\Theta^*_L$ [as an example for Apulia see Fig. 3(b)].

For establishing if the regional parameters must be estimated by $h_{24}$ or by daily rainfalls, in each subregion the empirical CDF of CV was compared with the one obtained by samples generated by the Monte Carlo technique. The analysis showed that 1,000 simulations are sufficient to obtain a robust estimate of the mean $\mu(CV)$ and standard deviation $\sigma(CV)$ of the coefficient of variation of generated samples. The analysis confirmed that only for the Campania region [Fig. 5(a)] could the ML estimates of the parameters $\Lambda^*$ and $\Theta^*$ be carried out by daily rainfall data.
empirical CDF of the standardized variable $y_1$. For $h_1$ rainfalls Tables 3 and 5 show the good agreement between historical statistics and generated ones.

Finally, for each subregion the following rough relationship of the growth curve is proposed:

$$ h_1 = a_1 + b_1 \ln T $$  \hspace{1cm} (16)

in which $a_1$ and $b_1$ = constants to be estimated by the least-squares method. Table 6, in which $a_1$ and $b_1$ are listed for each investigated subregion, shows two sets of numerical constants (for $T \leq 100$ years and $T > 100$ years for Calabria only. In Fig. 10, for the Calabrian C subregion, the comparison between (6) and (16) is plotted.

**APPLYING RDD RELATIONSHIP**

According to the developed regional statistical analysis, for Basilicata the RDD relationship is always (14), whereas for the other three regions (13) has to be applied only for $t = 1$ h.

Using (15) and (16), the rainfall-duration-frequency factor $f(1, T)$ has the following expression:

$$ f(1, T) = \frac{a_1 + b_1 \ln T}{a_{24} + b_{24} \ln T} $$  \hspace{1cm} (17)

Fig. 11 shows that $f(1, T)$ is strongly dependent on geographical factors. For applying (13) and (14) at ungauged sites,
Figs. 12 and 13 provide the iso-$\mu_{24}$ and iso-$n$ maps for each investigated region.

CONCLUSIONS

Most of the methods applied for constructing synthetic design storms are based on using an RDD equation; in other words, the knowledge of the rainfall depth $h_T$ occurring in a period of $t$ hours and having a return period of $T$ years is necessary. The estimate of this rainfall depth needs the choice of the theoretical cumulative distribution function to be used and the estimation of its parameters.

In recent years, the Italian National Research Group for the Prevention of Hydro-Geological Disasters developed the VAPI research project, which aimed at unifying the analysis of the annual rainfall series and flood series measured in Italy. The VAPI project is based on the use of the TCEV distribution and its hierarchical regionalization procedure. This choice is due to the ability of a TCEV distribution to generate data having a skewness coefficient sufficiently variable to account for the separation effect.

At first, after a brief review of the TCEV and the hierarchical regionalization estimating procedure of its parameters, a new RDD relationship, applicable for both the DD [(13)] and the DI [(14)] case, was deduced. For a given site, (13)
shows that for estimating \( h_{rT} \), the frequency distribution of the rainfall depth \( h_{r} \) has to be studied and the rainfall-duration-frequency factor \( f(t, T) \) and \( n \) coefficient, appearing in the power equation linking the mean \( \mu_{c} \) of the TCEV distribution with the duration \( t \) are necessary. If the DI hypothesis is used, \( f(t, T) \) is always equal to one.

Then, for each investigated geographical region (Apulia, Basilicata, Calabria, and Campania), the hierarchical regionalization procedure of TCEV parameters estimation was developed for the \( h_{36} \) and \( h_{1} \) rainfalls. For establishing if the regional parameters should be estimated by \( h_{36} \) or daily rainfalls, the ability of the TCEV distribution to reproduce the empirical CDF of the coefficient of skewness and variation of \( h_{36} \) samples was also controlled. The analysis established that only for the Campania region can the maximum likelihood estimates of the regional parameters \( A^{*} \) and \( \Theta^{*} \) be carried out by daily rainfall data. At the second level of the regionalization procedure, according to previous studies, Apulia and Campania were considered homogeneous regions, whereas Basilicata and Calabria were divided into homogeneous subregions. Because the probability distribution function \( F(h_{r}^{*}) \) is implicit for \( h_{r}^{*} \), for each subregion and \( t = 24 \text{ h} \) the growth curve was approximated by the rough relationship (15). For each investigated subregion the numerical values of \( a_{36} \) and \( b_{36} \) constants appearing in (15) were estimated by the least-squares method.

The statistical analysis of \( h_{1} \) rainfalls was developed only for the three regions (Apulia, Calabria, and Campania), for which the DD hypothesis was recognized. The analysis showed the ability of the TCEV distribution, with ML parameter estimates obtained from the \( h_{1} \) rainfall samples, to describe the empirical CDF of the standardized variable \( y_{i} \). For each subregion the growth curve was approximated by (16), in which \( a_{1} \) and \( b_{1} \) constants were estimated by the least-squares method.

Finally, for the investigated Italian geographical regions, the statistical analysis of \( h_{36} \) and \( h_{1} \) rainfalls allowed the calibration of the rainfall-duration frequency factor appearing in the RDD relationship. For each region, isoline maps of the mean rainfall with a duration \( t = 24 \text{ h} \) and of the exponent \( n \), useful for determining \( \mu_{36} \) and \( n \) estimates at ungauged sites, were also provided.

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APPENDIX. REFERENCES

