Il modello IS-LM: la sua derivazione algebrica

We saw that the market for goods and services can be described as follows:

$$C = \bar{C} + c(Y - T) - ar \qquad (1)$$

$$I^P = \bar{I} - br \qquad (2)$$

$$PAE = C + I^P + \bar{G} + \overline{NX} \qquad (3)$$

$$Y = PAE \qquad (4)$$

Equation (1) is an extended consumption function, Equation (2) relates planned investment to the real rate of interest *r*, Equation (3) defines planned aggregate expenditures and Equation (4) is the equilibrium condition in the market for goods and services. As in the text we shall continue to assume that inflation is zero so that the real rate of interest *r* equals the nominal rate *i*. Substituting for *C* and *IP* in Equation (3) gives

$$PAE = [\bar{C} - c\bar{T} + \bar{I} + \bar{G} + \overline{NX}] - (a + b)i + cY$$

or:

$$PAE = \bar{A} - fi + cY$$

where *A*– = autonomous expenditures and *f* = (*a* + *b*), which measures the responsiveness of consumption and investment expenditures to changes in the rate of interest. Substituting for *PAE* in the equilibrium condition (4) and collecting the terms in *Y* gives:

$$Y = \left(\frac{1}{1 - c}\right)[\bar{A} - fi] \qquad (5)$$

Because Equation (5) defines (*i, Y*) combinations which give equilibrium in the market for goods and services (*Y = PAE*) it is the equation for the IS curve. To derive the slope of the IS curve we can rewrite Equation (5) as:

$$i = \left(\frac{1}{f}\right)\bar{A} - \left(\frac{1 - c}{f}\right)Y \qquad (6)$$

Letting the Greek letter delta, or D, denote the phrase 'change in', then, for a constant level of autonomous expenditure *A*–:

$$\Delta i = -\left(\frac{1-c}{f}\right)\Delta Y$$

and the slope of the IS curve is:

$$\left(\frac{\Delta i}{\Delta Y}\right)_{IS} = -\left(\frac{1-c}{f}\right) \qquad (7)$$

Hence given the value of the marginal propensity to consume *c* the slope of the IS curve depends on the parameter *f*, which measures the response of consumption and investment to the rate of interest. Other things being equal, the greater is *f*, or the greater the responsiveness of consumption and investment to interest rate changes, the smaller the slope and the flatter the IS curve.

Example The IS curve
In a certain economy, c = 0.8, f = 1,000 and A– = 1,010. Derive the IS curve when i = 0.05, or 5 per cent, and when i = 0.01, or 1 per cent.
Using Equation (2) in Box for:

$$i = 0.05 : Y = \left(\frac{1}{0.2}\right)[1{,}010 - 1{,}000(0.05)] = 4{,}800$$
$$i = 0.01 : Y = \left(\frac{1}{0.2}\right)[1{,}010 - 1{,}000(0.01)] = 5{,}000$$

Point C corresponds to an (i, Y) combination (0.05, 4,800) and point D to a combination (0.01, 5,000). As both combinations give equilibrium in the market for goods and services both lie on the IS curve.

Exercise

In Euroland the components of planned aggregate expenditure are given as:

$$C = \overline{C} + c(Y - T) - ai$$
$$I^P = \overline{I} - bi$$
$$\overline{G} = 200 \quad T = 320 \quad NX = 0$$

If C– = 1,245, I– = 310, a = 1,000, b = 500 and c = 0.75, derive the equation for Euroland's IS curve and find the equilibrium values of Y when i = 0.01 and when i = 0.03.